Technical Note

Inconsistent Definitions of Pressure-Coupled Responses and Admittances of Solid Propellants

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Nomenclature

A_b	=	admittance of the propellant surface
a	=	speed of sound
M	=	Mach number
m	=	mass flow rate
p	=	pressure
Rp	=	pressure-coupled response
r	=	burn rate
T	=	temperature
u	=	velocity
γ	=	ratio of specific heats
ρ	=	gas density
ρ_{solid}	=	density of the solid propellant

Superscripts

ˆ	=	unsteady quantity
–	=	mean quantity

Introduction

WHEN an acoustic wave is present in a solid-propellant combustion environment, the mass flux from the combustion zone oscillates at the same frequency as the acoustics. The acoustic wave is either amplified or attenuated by the response of the combustion to the acoustic disturbance. When the acoustic wave is amplified, this process is called combustion instability. The amplification is quantitatively measured by a response function.

The ability to predict combustion stability for a solid-propellant formulation is essential to the formulator to prevent or minimize the effects of instabilities, such as an oscillatory thrust. Unfortunately, the prediction of response values for a particular propellant remains a technical challenge.

Most predictions of the response of propellants are based on test data, but there are a number of questions about the reliability of the standard test method, the T-burner. Alternate methods have been developed to measure the response of a propellant, including the ultrasound burner, the magnetic flowmeter, and the rotating valve

burner, but there are still inconsistencies between the results obtained by these different methods.

Aside from the experimental differences, the values of the pressure-coupled responses obtained by different researchers are often compared erroneously, for the simple reason that inconsistencies in the definitions of the responses and admittances are not considered. The use of different definitions has led to substantial confusion since the first theoretical treatments of the problem by Hart and McClure in 1959 (Ref. 1). The definitions and relations derived here are an attempt to alleviate this problem.

Definitions

In general, a response function characterizes an unsteady property that results from another unsteady property. In the case of the pressure-coupled response, the response is a measurement of an unsteady property that is caused by an unsteady gas pressure above the surface of the propellant. Traditionally, the resultant unsteady property is the mass flux. However, it has also been defined as a combination of the mass flux and the gas temperature. There is some confusion in the literature about these definitions and the correct relations between the pressure-coupled response and the admittance of the propellant surface.

The most common definition of the response is the ratio of the nondimensionalized mass flux to the nondimensionalized pressure oscillation. This first definition is

$$Rp_1 = (\hat{m}/\bar{m})/(\hat{p}/\bar{p}) \tag{1}$$

A less common definition of the pressure-coupled response includes the gas temperature,[2]

$$Rp_2 = \frac{\hat{m}/\bar{m} + \hat{T}/\bar{T}}{\hat{p}/\bar{p}} \tag{2}$$

The admittance is generally defined as a relation between the oscillatory velocity and the oscillatory pressure, as

$$A_{b1} = \bar{\rho} a(\hat{u}/\hat{p}) = (\hat{u}/a)/(\hat{p}/\gamma \bar{p}) \tag{3}$$

This definition has been used extensively in the literature since its first use by Culick.[3]

To arrive at the relation between the admittance and the definition of the response in Eq. (1), it is first necessary to linearize the mass flux in the gaseous phase and the adiabatic relation, as shown in Eqs. (4) and (5), respectively. If these two relations and the definition of the admittance are then substituted into the first definition of the pressure-coupled response, the relation shown in Eq. (6) is found. Thus,

$$\hat{m}/\bar{m} = \hat{u}/\bar{u} + \hat{\rho}/\bar{\rho} \tag{4}$$

$$\hat{\rho}/\bar{\rho} = (1/\gamma)(\hat{p}/\bar{p}) \tag{5}$$

$$Rp_1 = (A_{b1} + M)/\gamma M \tag{6}$$

The definition of the response that includes the temperature fluctuation has a slightly different relation to the admittance. The relation is obtained by linearizing mass continuity in both the solid and gaseous phases and the ideal gas law. The mass flux of the solid

phase in Eq. (7) is equal to the density of the solid propellant multiplied by the burn rate. When this is linearized, the expression in Eq. (8) is obtained, that is,

$$\bar{m} = \rho_{\text{solid}}\bar{r} \qquad (7)$$

$$\hat{m}/\bar{m} = \hat{r}/\bar{r} \qquad (8)$$

The ideal gas law can also be linearized to obtain the expression in Eq. (9). If the linearized mass fluxes in both the gaseous and solid phases [Eqs. (4) and (8)] are combined and substituted into Eq. (9), then Eq. (10) is found:

$$\hat{p}/\bar{p} = \hat{\rho}/\bar{\rho} + \hat{T}/\bar{T} \qquad (9)$$

$$\hat{m}/\bar{m} + \hat{T}/\bar{T} = \hat{u}/\bar{u} + \hat{\rho}/\bar{\rho} \qquad (10)$$

When Eq. (10) is substituted back into Eq. (2), a different relation between the pressure-coupled response and the admittance is obtained, as

$$Rp_2 = 1 + A_{b1}/\gamma M \qquad (11)$$

Responses obtained from this relation are greater than the values obtained from Eq. (6) by $1 - 1/\gamma$ for the same value of the admittance. This difference is on the order of 0.1 for most solid propellants, a small but significant difference.

A second definition of the admittance also leads to a yet another relation between the admittance and Eq. 1. This definition of the admittance,

$$A_{b2} = (\hat{u}/\bar{u})/(\hat{p}/\bar{p}) \qquad (12)$$

has not been used with the definition of the pressure-coupled response. Culick defined the admittance as a function of the inverse of the acoustic impedance.[4] The relation between this definition of the admittance and Eq. (1) is derived similarly to Eq. (6). The gaseous mass flux and the adiabatic relation are linearized and substituted into the definition of the response. Substituting in the different definition of the admittance gives the expression

$$Rp_1 = A_{b2} + 1/\gamma \qquad (13)$$

This relation between the admittance and the response also produces significant differences in the value of the response for the same value of the admittance. In this case, the difference between the two responses is a factor of the admittance. Responses found from Eq. (6) could potentially be as much an order of magnitude greater than a response found using the relation in Eq. (13) for the same value of the admittance. Fortunately, this relation is no longer commonly used to calculate the pressure-coupled response, although this is the definition of the admittance that is most commonly used in acoustic theory.

Conclusions

Although the two different definitions of the pressure-coupled response have lead to some confusion, the second definition of the response that includes the temperature flux oscillation is not commonly used today. The potential for confusion lies primarily in the use of different definitions of the admittance. Several of the experimental techniques use a measurement of the admittance to derive a response. It is vital that the admittance and pressure-coupled response be defined for the purpose of comparison to other experimental techniques.

References

[1]Hart, R. W., and McClure, F. T., "Combustion Instability: Acoustic Interaction with a Burning Solid Propellant Surface," *Journal of Chemical Physics*, Vol. 30, No. 6, 1959, pp. 1501–1514.

[2]Micci, M., "Workshop Report: Methods for Measuring Solid Propellant Combustion Response," *23rd JANNAF Combustion Meeting*, 1986, pp. 51–57.

[3]Culick, F. E. C., "A Review of Calculations for Unsteady Burning of a Solid Propellant," *AIAA Journal*, Vol. 6, No. 12, 1968, pp. 2241–2255.

[4]Culick, F. E. C., "Calculation of the Admittance Function for a Burning Surface," *Acta Astronautica*, Vol. 13, No. 3, 1967, pp. 221–237.